

Model Validation via Uncertainty Propagation and Data Transformations

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Model validation has become a primary means to evaluate accuracy and reliability of computational simulations in engineering design. Because of uncertainties involved in modeling, manufacturing processes, and measurement systems, the assessment of the validity of a modeling approach must be conducted based on stochastic measurements to provide designers with confidence in using a model. A generic model validation methodology via uncertainty propagation and data transformations is presented. The approach reduces the number of physical tests at each design setting to one by shifting the evaluation effort to uncertainty propagation of the computational model. Response surface methodology is used to create metamodels as less costly approximations of simulation models for the uncertainty propagation. Methods for validating models with both normal and nonnormal response distributions are proposed. The methodology is illustrated with the examination of the validity of two finite element analysis models for predicting springback angles in a sample flanging process.

I. Introduction

THE increased dependence on using computer simulation models in engineering design presents a critical issue of confidence in modeling and simulation accuracy. Model verification and validation are the primary methods for building and quantifying confidence, as well as for the demonstration of correctness of a model.^{1,2} Briefly, model verification is the assessment of the solution accuracy of a mathematical model. Model validation, on the other hand, is the assessment of how accurately the mathematical model represents the real world application.³ Thus, in verification, the relationship of the simulation to the real world is not an issue, whereas in validation, the relationship between the virtual (computation) and the real world, that is, experimental data, is an issue.

One limitation of existing model validation approaches is that they are restricted to validation at a particular design setting. There is no guarantee that the conclusion can be extended over the entire design space. In addition, model validations are frequently based on comparisons between the output from deterministic simulations and output from single or repeated experiments. Existing statistical approaches, for which the physical experiment has to be repeated a sufficient number of independent times, is not practical for many applications, simply due to the cost and time commitment associated with experiments. Furthermore, deterministic simulations for model validation do not consider uncertainty at all. Although recent

model validation approaches propose to shift the effort to propagating the uncertainty in model predictions, which implies that a model validation should include all relevant sources of uncertainties, little work has been accomplished in this area.^{1,2,4} Because realistic mathematical models should contemplate uncertainties, the assessment of the validity of a modeling approach must be conducted based on stochastic measurements to provide designers with confidence in using a model.

Traditionally, a model has been considered valid if it reproduces the results with adequate accuracy. The two traditional model validation approaches are 1) subjective and 2) quantitative comparisons of model predictions and experimental observations. Subjective comparisons are through visual inspection of x - y plots, scatter plots, and contour plots. Although they show the trend in data over time and space, subjective comparisons depend on graphical details. Quantitative comparisons, including measures of correlation coefficient and other weighted and nonweighted norms, quantify the “distance” but become very subjective when defining what magnitudes of the measures are acceptable. To quantify model validity from a stochastic perspective, researchers have proposed various statistical inference techniques, such as χ^2 test on residuals between model and experimental results.⁵ These statistical inferences require multiple evaluations of the model and experiments, as well as many assumptions that are difficult to satisfy. Therefore, there is a need for a model validation approach that uses the least amount of statistical assumptions and requires the minimum number of physical experiments.

In this paper, we present a rigorous and practical approach for model validation (model validation via uncertainty propagation) that utilizes the knowledge of system variations along with computationally efficient uncertainty propagation techniques to provide a stochastic assessment of the validity of a modeling approach for a specified design space. Various sources of uncertainties in modeling and in physical tests are evaluated, and the number of physical testing at each design setting is reduced to one. Response surface methodology is used to create a metamodel of an original simulation model, and therefore, the computational effort for uncertainty propagation is reduced. By the employment of data transformations, the approach can also be applied to the response distributions that are nonnormal. This helps us represent the data in a form that satisfies the assumptions underlying the r^2 method, an approach used in this work to determine whether the results from physical experiments

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fall inside or outside of the prespecified confidence region. Although the proposed methodology is demonstrated for validating two finite element models (FEMs) for simulating sheet metal forming, namely, a flanging process, it can be generalized to other engineering problems.

This paper is organized as follows. In Sec. II, the technical background of this research is provided. The major types of uncertainties in modeling are first introduced and classified into three categories. Existing techniques on uncertainty propagation are then reviewed, and the background of the response surface methodology and statistical data transformations are provided. Our proposed model validation approach is described in Sec. III. In Sec. IV, our proposed approach is demonstrated using a case study in sheet metal forming by examining two finite element-based models. Finally, conclusions are provided in Sec. V.

II. Technical Background

A. Classification of Uncertainties

Various types of uncertainties exist in any physical system and in its modeling process and can affect the final experimental or predicted system response. Different ways of classifying uncertainties may be found in the literature.^{6–9} In this work, we classify uncertainties into three major categories:

Type 1 is uncertainty that is associated with the inherent variation in the physical system or environment that is under consideration, for example, uncertainty associated with incoming material, initial part geometry, tooling setup, process setup, and operating environment.

Type 2 is uncertainty that is associated with deficiency in any phase or activity of the simulation process that originates in lack of system knowledge, for example, uncertainty associated with the lack of knowledge in the laws describing the behavior of the system under various conditions, etc.

Type 3 is uncertainty that is associated with error that belongs to recognizable deficiency but is not due to lack of knowledge, for example, uncertainty associated with the limitations of numerical methods used to construct simulation models.

When the stochastic assessment of model validity is provided, all three types of uncertainties should be taken into account.

B. Techniques for Uncertainty Propagation

The use of an analysis approach to estimate the effect of uncertainties on model prediction is referred to as uncertainty propagation. Several categories of methods exist in the literature. The first category is the conventional sample-based approach such as Monte Carlo simulations (MCS). Although alternative sampling techniques such as quasi-Monte Carlo simulations including Halton sequence,¹⁰ Hammersley sequence,¹¹ and Latin supercube sampling¹² have been proposed, none of these techniques are computationally feasible for problems that require complex computer simulations, each taking at least a few minutes or even hours or days. Validating a modeling approach at multiple design settings becomes computationally infeasible. The second category of uncertainty propagation approach is based on sensitivity analysis. Most of these methods only provide the information of mean and variance based on approximations. The level of accuracy is not sufficient for applications in model validation. Here, we propose to use a response surface model (RSM) (or metamodel) to replace the numerical model for uncertainty propagation. The RSM is generated as a function of design variables and parameters. MCS are later performed using the RSM as a surrogate of the original numerical program. Details of response surface methodologies are provided in the next section.

C. Response Surface Methodologies

Response surface methodologies are well-known approaches for constructing approximation models based on either physical experiments or computer experiments (simulations).¹³ In this work, our interest is in the latter where computer experiments are conducted by simulating the to-be-validated model to build RSMs. They are often referred to as metamodels because they provide a “model of the model,”¹⁴ which replaces the expensive simulation models during the design and optimization process. In this paper, RSMs based

on simulation results from FEMs are constructed and tested for model validation by using two response surface modeling methods: polynomial regression (PR) and kriging (KG) methods.

PR models have been applied by a number of researchers^{15,16} to the design of complex engineering systems. In spite of the advantage obtained from the smoothing capability of PR for noisy functions, there is always a drawback when applying PR to model highly nonlinear behaviors. Higher-order polynomials can be used; however, instabilities may arise,¹⁷ or it may be too difficult to take sufficient sample data to estimate all coefficients in the polynomial equation, particularly in large dimensions.

A KG model¹⁸ postulates a combination of a polynomial model and a departure of the polynomial model, where the latter is assumed to be a realization of a stochastic process with a zero mean and a spatial correlation function. A variety of correlation functions can be chosen¹⁹; the Gaussian correlation function proposed in Ref. 18 is the most frequently used. The KG method is extremely flexible and can capture any type of nonlinear behavior due to the wide range of the correlation functions. The major disadvantages of the KG process are that model construction can be very time consuming and that they could be ill conditioned.²⁰

In our earlier works, the advantages and limitations of various metamodeling techniques have been examined by use of multiple modeling criteria and multiple test problems.^{21,22} Our strategy in this work is to first fit a second-order polynomial model. If the accuracy is not satisfactory, the KG method will be employed; otherwise, the low-cost polynomial model will be used for uncertainty propagation and model validation.

D. Data Transformations

Many statistical tests are based on the assumption of normality. When data deviate from normality, an appropriate transformation can often yield a data set that approximately follows a normal distribution.²³ Generally, response distributions obtained from uncertainty propagation at multiple design points may not be normal. Data transformations are, therefore, employed to use the proposed validation approach that is based on the normality assumption.

One of the most common and simplistic parametric transformation families studied by Tukey²⁴ and later modified by Box and Cox²⁵ is

$$Z^{(\lambda)} = \begin{cases} (Z^\lambda - 1)/\lambda, & \lambda \neq 0 \\ \log Z, & \lambda = 0 \end{cases} \quad (1)$$

where λ is the transformation parameter. For different values of λ , different transformations are obtained. When $\lambda = 1$, no transformation occurs. When $\lambda < 1$, the transformation makes the variance of residuals smaller at large Z and makes it larger at small Z . When $\lambda > 1$, it has the opposite effects of $\lambda < 1$. When $\lambda = 0$, the natural logarithm is used [Eq. (1)].

The Box–Cox transformation²⁵ in Eq. (1), called the power transformation, is only appropriate for positive data. Hinkley,²⁶ Manly,²⁷ John and Draper,²⁸ and Yeo and Johnson²⁹ proposed alternative families of transformations that can be used to compensate for the restrictions on Z , to obtain an approximate symmetry or to make the distribution closer to normal.

The parameters of a transformation, for example, λ , can be selected through a trial and error approach until good normal probability plots are obtained, through optimization based on maximum likelihood estimation or Bayesian estimation (see Ref. 25), likelihood ratio test,³⁰ or M-estimators,³¹ etc. Atkinson and Riani³² and Krzanowski³³ discussed aspects of multivariate data transformations in more detail.

It is often more useful to apply transformations of predictor (model input) variables, along with the transformations of the dependent (model output) variables. Box and Tidwell³⁴ provided an iterative procedure to estimate appropriate transformations of the original model inputs. Atkinson and Riani³² discussed different models and reasons for which transformations of predictor variables can be applied. Note that applying the existing transformation techniques may have little effect if the values of the response are far from zero and the scatter in the observations is relatively small. (In

other words, the ratio of the largest to smallest observation should not be too close to one.³⁰⁾

III. Proposed Model Validation Approach

A. General Description of the Approach

Our proposed model validation approach is shown in Fig. 1. The whole process includes four major phases, in which phase 2 and phase 3 can be implemented in parallel. Phase 1 is the problem setup stage. Here, uncertainties of all of the types described in Sec. II.A are investigated, and probabilistic descriptions of model inputs are established. With the aim of model validation over a design space rather than at a single design point, sample design settings, represented by x^i , $i = 1, \dots, n$, are formed using different combinations of values of design variables. The sampling can be based on the knowledge of critical combinations of design variables at different levels, the standard statistical techniques such as design of experiments,¹³ or other methods for efficient data sampling, for example, optimal Latin hypercube.³⁵ These techniques will be useful in reducing the size of samples when the number of design variables considered is large.

Phase 2 is the (physical) experimental stage. One of the cornerstones of this proposed approach is the minimum number of physical tests required. Physical experiments will be performed only once at each design setting identified in phase 1. Measurements are taken for the model responses that are of interest. The results of experiments are denoted as Y^i , $i = 1, \dots, n$. Errors of measurements are predicted.

Phase 3 (model uncertainty propagation) is the stage for uncertainty propagation based on the to-be-tested (computational) model. For computationally expensive models, we propose to first construct a RSM based on samples of numerical simulation results. Techniques introduced in Sec. II.C can be applied here. Next, the total uncertainty of the response prediction is analyzed using the response surface model through MCS introduced as follows. Note that for model validation at multiple design points, uncertainty of the response prediction needs to be evaluated for each of the design points as identified in phase 1.

The uncertainty of the (computational) model prediction can be evaluated by uncertainty propagation using MCS applied to the metamodel (in this case, RSM), following the uncertainty descriptions identified in phase 1. When a sufficient number of simulations are performed, the MCS is robust in the sense that it provides good estimates of uncertainty in the predicted parameters, whether the model is highly nonlinear or not. The MCS also provides estimates of the shape of the probability density functions (PDF), which are used further in phase 4 for model validation. If the normality checks for the PDFs from MCS are rejected, we propose to apply data transformations to data from the simulation models before constructing

response surface models so that the transformed distributions become normal.

Phase 4 is the model validation phase, when the stochastic assessment of model validity is drawn based on the comparisons of the physical experimental results from phase 2 and the computational results from phase 3. The strategies introduced by Hills and Trucano¹ are followed here. Because their validation criterion for multiple design settings is applicable only for normal response distributions, data transformation is proposed in this work to extend the applicability of the proposed approach to nonnormal response distributions. Details of model validation strategies for single and multiple design points, procedures for data transformation, and accounting various sources of errors are discussed next.

B. Model Validation at a Single Design Point

The Hills and Trucano¹ method states that, for a given confidence bound, for example, $100 \times (1 - \alpha\%)$, if the physical experiment falls within the performance range obtained from the computer model (here the PDF obtained from the MCS in phase 3), it indicates that the model is consistent with the experimental result; however, we can not say the model is valid for the confidence bound. On the other hand, if one physical experiment is outside of the performance range, then we would reject the model for that specified confidence bound $[100 \times (1 - \alpha\%)]$. Our strategy of model validation is to identify at which critical limit of confidence level (p value) the physical experiment falls exactly at the boundary of the performance range obtained from the computer model (Fig. 2). Therefore, if the given confidence level is lower than the critical limit of confidence level, the model will be rejected, and vice versa.

As shown in Fig. 2, the PDF describes the distribution of a response based on the (computational) model for the given uncertainty description at a single design point. The confidence limit with which one cannot reject the simulation model is the area under the PDF curve that bounds exactly on the physical experiment, includes the mean of the PDF, and excludes the two equally sized tails that depend on the location of the physical experiment. If the confidence limit is identified as $\gamma\%$, which is smaller than the given confidence bound, for example, $100 \times (1 - \alpha\%)$ in Fig. 2, we cannot reject the model for an experiment that falls on the boundary of $\gamma\%$; otherwise we can reject the model because the physical experiment falls outside the distribution range. When a model is rejected, it indicates that a new model needs to be constructed, and the whole procedure of model validation should be carried out again. Note that because stochastic assessments are provided for model validity, there are certain risks associated with the error of hypothesis testing.³⁶ In our case, the false positive error (commonly referred to as a type 1 error) is the error of rejecting a model although the true state is that the model is indeed valid. The probability of leading to this outcome is $\alpha\%$. We note that providing a higher confidence bound (lower

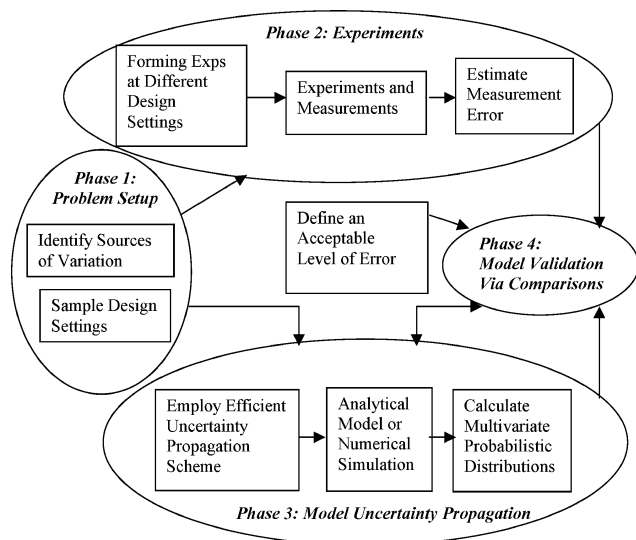


Fig. 1 Procedure for model validation.

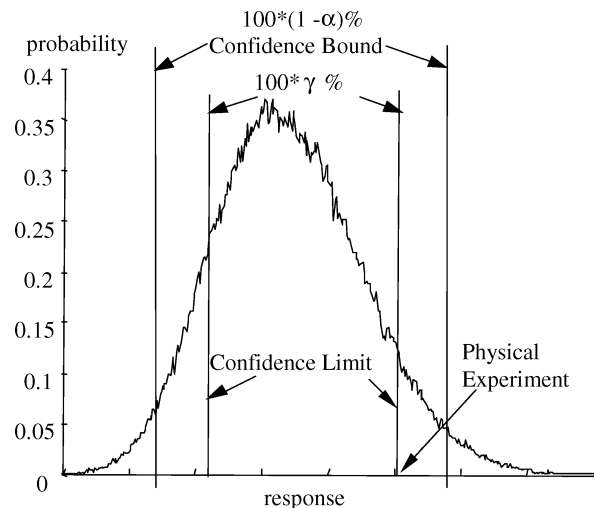


Fig. 2 Model validation for a single design point.

$\alpha\%$) would widen our acceptance region. Whereas it will reduce our chances of rejecting a valid model, it would also increase our chance of accepting an invalid model, that is, increase the probability of making the false negative error (referred to as type 2 error). Indications of type 1 and type 2 errors in model validation were discussed by Oberkampf and Trucano,² where they related type 1 error to a model builder's risk and type 2 error to model users' risk.

C. Model Validation at Multiple Design Settings

When a model needs to be validated at multiple design settings, the experimental results need to be compared against the joint probability distributions of a response at multiple design settings. The probability distributions of y^i at multiple design settings n are used to generate the joint probability distributions (multidimensional histogram). The contours of the joint probability distributions are used to define the boundary of a given confidence level for model validation and are compared with the results from physical tests. An example of model validation for a problem with two physical tests (corresponding to two design settings) is provided in Fig. 3. The joint PDF of y^1 and y^2 is first obtained for the same response, and then the boundary with $1 - \alpha$ confidence level is determined by the isocount contour that contains $100(1 - \alpha)\%$ samples of MCS conducted over the RSM. Theoretically, if the experimental result Y in an n -dimensional space ($n = 2$ in this example) falls within the boundary, it indicates that we cannot reject the model with a confidence level of $(1 - \alpha)$. If the point falls outside of the boundary, then we can reject the model with a confidence level of $(1 - \alpha)$. Note the results of single experiments at multiple design settings now become a single point in the multivariate histogram space (Fig. 3).

For multivariate distributions symmetric about their means, contours of constant probability are given by ellipses determined with r^2 . Note that r^2 , which can be thought of as a square of the weighted distance of the physical experiments from the multivariate mean, can be related to normal probability through the chi-square distribution for $100 \times (1 - \alpha)\%$ confidence with n degrees of freedom (n design settings). The prediction model can be rejected at $100 \times (1 - \alpha)\%$ if the combination of multiple design points measured from physical experiments is outside of $100 \times (1 - \alpha)\%$ confidence region.

According to Hills and Trucano¹ a constant probability is given by the following ellipses where r^2 is constant for isoproability curves:

$$r^2 = [y_1 - y_{\text{mean}1} \quad y_2 - y_{\text{mean}2} \quad \cdots \quad y_n - y_{\text{mean}n}] \times V^{-1} \begin{bmatrix} y_1 - y_{\text{mean}1} \\ y_2 - y_{\text{mean}2} \\ \vdots \\ y_n - y_{\text{mean}n} \end{bmatrix} \quad (2)$$

In Eq. (2), y_i , $i = 1, \dots, n$, is for the single experimental result for each design setting i , and $y_{\text{mean}i}$ is the mean of the random samples

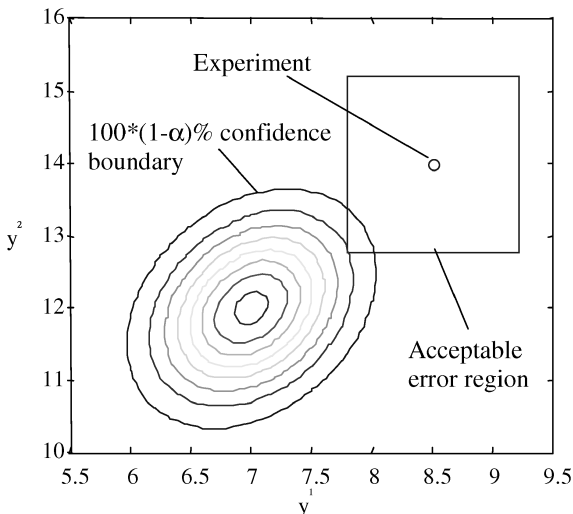


Fig. 3 Model validation at two design settings.

obtained from the computer model at each testing point i . The V matrix is the n by n covariance matrix based on the random samples.

For model validation, the critical value of r^2 is obtained as

$$r_{\text{critical}}^2 = l_{1-\alpha}^2(n) \quad (3)$$

where l is the value associated with the $100 \times (1 - \alpha)\%$ confidence for n testing points through the chi-square distribution. If the value of r^2 from Eq. (2) is less than the critical value of r^2 from Eq. (3), then we do not possess statistically significant evidence to declare our model invalid and vice versa. When an acceptable error region is considered (box in Fig. 3), the value of r^2 is calculated based on the location of the extreme corner of the box.

D. Data Transformations for Model Validation Purposes

As stated earlier, the strategies for model validation introduced by Hills and Trucano¹ are followed in this research. For comparison purposes, a test statistic r^2 is employed. To apply the r^2 criterion for model validation, the assumption of normality of the multivariate joint probability distributions has to be satisfied.

Multivariate normality is the assumption that all dimensions and all combinations of the dimensions are normally distributed. When the assumption is met, the residuals (differences between predicted and obtained response values) are symmetrically distributed around a mean of zero and follow a normal distribution. The assumption of normality often leads to tests that are simple, mathematically tractable, and powerful compared to tests that do not make the normality assumption.

The two methods for normality screening are the statistical approach and the graphical approach. The statistical method employs examinations of significance for skewness and kurtosis. Mardia³⁷ suggested useful measures of skewness and kurtosis. Skewness is related to the symmetry of the distribution, whereas kurtosis is related to the peakedness of a distribution, either too peaked or too flat. The graphical method visually assesses the distributions of the data and compares them to the normal distribution.

Transformations can be applied to both the response (model output) and predictor (model input) variables following the approaches discussed in Sec. II.D. Only after employing transformations can we apply the model validation procedure described earlier in Sec. III. RSM, MCS, and Eqs. (2) and (3) are applied for the transformed model to assess the model validity.

E. Measurement Error, RSM Error, and Acceptable Level of Error

In the proposed model validation procedure, it is also important to consider various uncertainties (errors) that cannot be predicted by the uncertainty propagation based on the computational model. These errors include the measurement errors, the response surface model error, and the acceptable level of error. To simplify the process, we count the measurement errors and response surface model error by including them directly to the prediction uncertainty obtained through uncertainty propagation. Specifying an acceptable level of error is practically significant because the discrepancy between the simulated and experimental results indicates the errors associated with the model structure and numerical procedures (type 2 and 3 uncertainties discussed in Sec. II). Approximated models should not be declared invalid if they provide predictions within an error that the user finds acceptable for a particular application. The acceptable level of error of a modeling approach is modeled as a box or a circle around the physical test point in Fig. 3. Figure 3 shows a situation in which the confidence region of the model prediction and the acceptable error region overlap. This indicates that we cannot declare that the model is invalid for the given confidence level considering the acceptable level of error.

IV. Validating Finite Element Model of Sheet Metal Flanging Process

A. Sheet Metal Flanging Process and Its Modeling

Sheet metal forming is one of the dominant processes in the manufacture of automobiles, aircraft, appliances, and many other

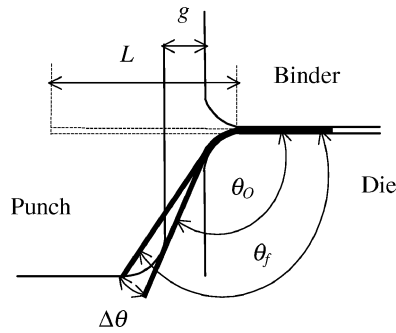


Fig. 4 Schematic of springback in flanging; g is gap between die and punch, θ_0 is flange angle at fully loaded configuration, θ_f is flange angle of unloaded configuration, and $\Delta\theta$ is springback.

products. As one of the most common processes for deforming sheet metals, flanging is used to bend an edge of a part to increase the stiffness of a sheet panel and/or to create a mating surface for subsequent assemblies. As the tooling is retracted, the elastic strain energy stored in the material recovers to reach a new equilibrium and causes a geometric distortion due to elastic recovery (Fig. 4), the so-called springback.³⁸ Springback refers to the shape discrepancy between the fully loaded and unloaded configurations as shown in Fig. 4.

Springback depends on a complex interaction between material properties, part geometry, die design, and processing parameters. The capability to model and simulate the springback phenomenon early in the new product design process can significantly reduce the product development cycle and costs. However, many factors influence the amount of springback in a physical test. Prediction and experimental testing of springback is particularly sensitive to the various types of uncertainties as discussed in Refs. 39 and 40. With reference to the definitions of the three types of uncertainties described in Sec. II.A, examples of type 1 uncertainty are the parameters related to incoming sheet metal material, initial geometry, and process setup. An example of type 2 uncertainty is that the hardening law to describe the behavior of sheet metal under loading and reverse loading is often uncertain in material characterization. An example of type 3 uncertainty is the numerical error caused by using different finite element analysis methods for springback angle estimation, for example, implicit finite element method, explicit finite element method, etc.

Various modeling approaches have been used to model the flanging process. These models include both analytical models and finite element analysis-based models. In this study, we illustrate how the proposed model validation approach can be applied to validate two finite element analysis models that model the blank plasticity with the combined hardening (model 1) and isotropic hardening (model 2) laws.⁴¹ The process is modeled by using an implicit and static non-linear finite element code, ABAQUS/standard (version 5.8.). The two models with combined hardening and isotropic hardening laws are used to illustrate the effect when the data from MCS follow either normal or a nonnormal distribution. Normalizing transformations are applied to the data from the model with the isotropic hardening law for the model validation procedure. The angle at the fully unloaded configuration (Fig. 4) is considered as the process output (the response).

B. Problem Setup, Experiments, and Uncertainty Propagation in Validating Sheet Metal Forming Process Models

We illustrate in this section how the major phases in the proposed model validation approach are followed for our case study.

Phase 1: Problem Setup

To accomplish phase 1, design variables and design parameters that affect the process output (final flange angle θ_f) are determined. Primarily, two design variables that are related to the process setup are considered, that is, flange length L and gap space g and design parameters that are related to the material are selected, that is, sheet

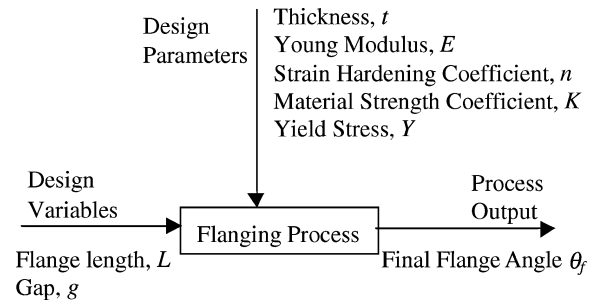


Fig. 5 System diagram for flanging process.

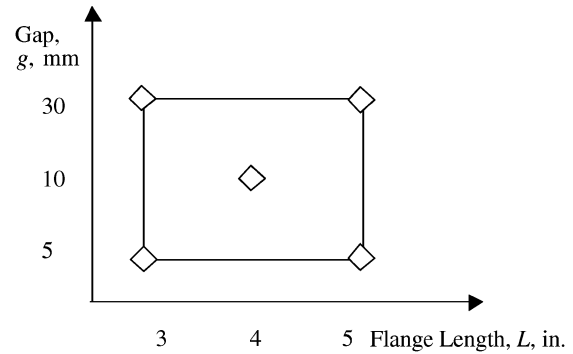


Fig. 6 Sample design settings of flanging process for model validation.

thickness t and material properties, namely, Young's modulus E , strain-hardening coefficient n , material strength coefficient K , and yield stress Y (Fig. 5). Design parameters are uncontrollable (given), whereas design variables can be controlled over the design space to achieve the desired process output.

To form sample design settings, different combinations of values of design variables, that is, L and g , are used. Five sample design settings are formed with combinations of low and high levels of flange length (3 and 5 in.; 76.2 and 127.0 mm) and gap (5 and 30 mm) plus a design point close to the middle (4 in. and 10 mm; 101.6 and 10 mm) (Fig. 6). These values for low, middle, and high levels of flange length and gap are selected so that they can cover the whole design space as uniformly as possible.

The variations of design variables and design parameters are identified in this phase. Based on experimental data, obtained by tensile tests, the relationships among K , n , and Y for carbon steel sheet metals used in the tests are approximated as

$$K(n) = 1128.5n + 499.97 \quad (4)$$

$$Y(n) = -779.8n + 484.85 \quad (5)$$

Therefore, for the four parameters describing the material property, two are independent parameters (n and E) and the other two (K and Y) are dependent. Also, the statistical descriptions for these material parameters are obtained. The distribution of Young's modulus E is assumed to be a normal distribution with 197949.7 and 12914.7 MPa as the mean and standard deviation, respectively. The distribution of strain-hardening exponent n is assumed to be a uniform distribution from 0.10 to 0.18. The distributions of the strength coefficient K and yield stress Y depend on n as shown in Eqs. (2) and (3). The distribution of sheet thickness t is assumed to be a normal distribution with 1.5529 and 0.0190 mm as the mean and standard deviation, respectively. Similarly, the variation of the design variable gap space g is assumed to be normally distributed with a standard deviation of 0.6 mm; note that the mean of the gap will change based on the location of the design point. The variation of flange length L is ignored because the flanging accuracy tolerance is insignificant comparing to the effect of the other change on final flange angle.

Phase 2: Experiments and Measurements

In this phase, physical experiments are conducted and measurement errors are estimated. The dimensions of the sheet blank used

in the physical experiments are 203.2×203.2 mm (8×8 in.). The flanging process uses a punch, a binder, a draw die, and a blank. The experiments have been performed by the 150-ton computer-controlled HPM hydraulic press in the Advanced Materials Processing Laboratory at Northwestern University. The unloaded configurations, that is, the angles between two planes in degrees (Fig. 4), have been measured by a coordinate measuring machine (Brown and Sharpe MicroVal Series Coordinate Measuring Machine B89) in the metrology laboratory at Northwestern University.

Phase 3: Model Simulation and Uncertainty Propagation

The flanging process has been numerically simulated based on two FEMs; namely, model 1 uses the combined hardening law to the sheet material and model 2 uses the isotropic hardening law.

The process has been modeled by an implicit and static nonlinear commercial finite element code, ABAQUS/Standard.⁴¹ There were 1440 8-node, two-dimensional (plane strain) continuum elements with reduced integration used in this problem to model the sheet blank (ABAQUS element type CPE8R). The sheet thickness was modeled with six layers. Tools were modeled as rigid surfaces. The coefficient of friction is set to 0.125. The interface between the tooling and the sheet was modeled by interface elements (ABAQUS element type IRS22), and the penalty-based contact algorithm was used. For a better convergence rate, the surface interaction is modeled by a soft contact. The analysis is performed in six steps: moving the binder toward the blank, developing the binder force, moving the punch down to flange the blank, retracting the punch up, releasing the binder force, and, finally, moving the binder up.

The following two cases are considered in the simulation experiments for creating the RSMs for model validation. The procedure is illustrated here only with model 1 (combined hardening law). A similar procedure is followed in validating model 2 (isotropic hardening law).

Case 1: validation at a single design point. There were 81 simulation experiments conducted to create a RSM for model validation at a single design point (3, 30), that is, flange length at 3 in. and gap at 30 mm. The RSM represents the springback angle as a function across over a range of design parameters g , t , E , n , K , and Y corresponding to a single design setting of $L = 3$ in. The 81 simulation experiments are designed based on various combinations of g , t , E , n , K , and Y , where three levels are considered for both gap ($g = 25$, 30, and 35 in. at each level) and thickness ($t = 1.483$, 1.545, and 1.608 mm at each level) and a full factorial design of these two factors are combined with nine settings of E , n , K , and Y that capture a wide range of the material properties.

Case 2: validation for multiple design settings. There were 243 simulation experiments conducted to create the RSM for model validation at five design points, that is, the following combinations of design variables (flange length in inches and gap in millimeters): (3, 5), (3, 30), (4, 10), (5, 30), and (5, 5). The response surface model represents the final flange angle as a function of L , g , t , E , n , K , and Y . The 243 simulation experiments are designed based on various combinations of L , g , t , E , n , K , and Y . Similar to the strategy used for designing the experiments in case 1, three levels are considered for flange length L , gap g , and thickness t and a full factorial design of these three factors are combined with nine settings of E , n , K , and Y that capture a wide range of the material properties.

The second-order PR approximation models are first used to create RSMs for both cases 1 and 2. The accuracy is assessed by examining the sum of squares of error (SSE) based on a set of confirmation tests. For model 1, the results are obtained as SSE for PR is 0.0212 for case 1 and SSE for PR is 0.9862 for case 2. When it is considered that the magnitude of the angle of the final configuration is in the range of 100–150 deg, the achieved SSE from PR is quite satisfactory. Therefore, for uncertainty propagation and model validation, the results from the polynomial models will be used.

Once the RSMs are created, the MCS has been used to predict efficiently the distributions of the final flange angle under uncertainty using 200,000 random sample points. The uncertainty descriptions identified in phase 1 are followed for random sampling.

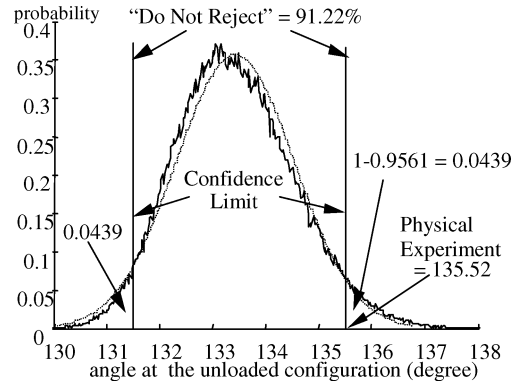


Fig. 7 Confidence limits based on polynomial model at single design point (3, 30), model 1; lightface curve is PDF of fitted normal distribution.

The predicted distributions of the final flange angle will be presented together with the validity results next.

Phase 4: Model Validation via Comparisons

Normality check. To simplify the model validation process, the predicted distributions of the final flange angle (for a single design point and each individual design point in multiple design settings) have been checked for normality. The resulting probability distributions from model 1 are plotted in Figs. 7 (case 1) and 8 (case 2). In Figs. 7 and 8, the light face PDF curve is the fitted normal distribution. Note that, in general, the predictions (considered separately for each design point) based on polynomial models are all very close to normal. A Kolmogorov–Smirnov (K–S) test (see Ref. 42) has been conducted for the normality check. Following procedures in the literature, the sample size for the K–S test is determined to be $N = 1000$ (Ref. 42, p. 431). Then 1000 samples are randomly selected from the 200,000 simulations. If the K–S statistic obtained from a K–S test is greater than the critical value, here 0.043 for $N = 1000$, the test rejects the null hypothesis (which states that the sample is drawn from a normal distribution). The K–S statistic is obtained as 0.04 for case 1, which means we cannot reject the null hypothesis at $\alpha = 0.05$, and therefore, the distribution can be considered as normal. The normality assumption can greatly simplify the validation process, which is introduced next.

Validation of model 1 (combined hardening law). For the single design point (3, 30) in case 1, the results of the predicted springback angles based on MCS using the response surface model are compared with the result from a single physical experiment. As shown in Fig. 7, the angle obtained from the experiment is 135.52 deg, and 95.61% of the angles predicted with simulation based on the polynomial model are smaller than the value of 135.52 (the left tail with the middle do not reject area in Fig. 7 together are $0.0439 + 0.9122 = 0.9561$). Thus, $[0.9561 - (1 - 0.9561)] = 0.9122$ (the do not reject area in Fig. 7) is the confidence level with which one cannot reject the simulation model. The two tails (each equal to $1 - 0.9561 = 0.0439$) are the reject the model area.

Based on the identified critical confidence limit, we can say that if the confidence level is given at 90% ($< 91.22\%$), we can reject the model. If the confidence level is given at 95%, we cannot reject the model. We note that providing a higher confidence level, for example, 99%, would widen our acceptance region; although it will reduce our chances of rejecting a valid model, it would also increase our chance of accepting an invalid model.

For case 2, from the results of normality check conducted earlier, it is assumed that the total model uncertainty for five design points could be modeled by jointly distributed normal PDFs. One physical experiment at each design point has been considered (Fig. 8). The angles obtained from the experiments at each design point are 134.9287, 106.5019, 111.6919, 135.2204, and 106.7697 at design

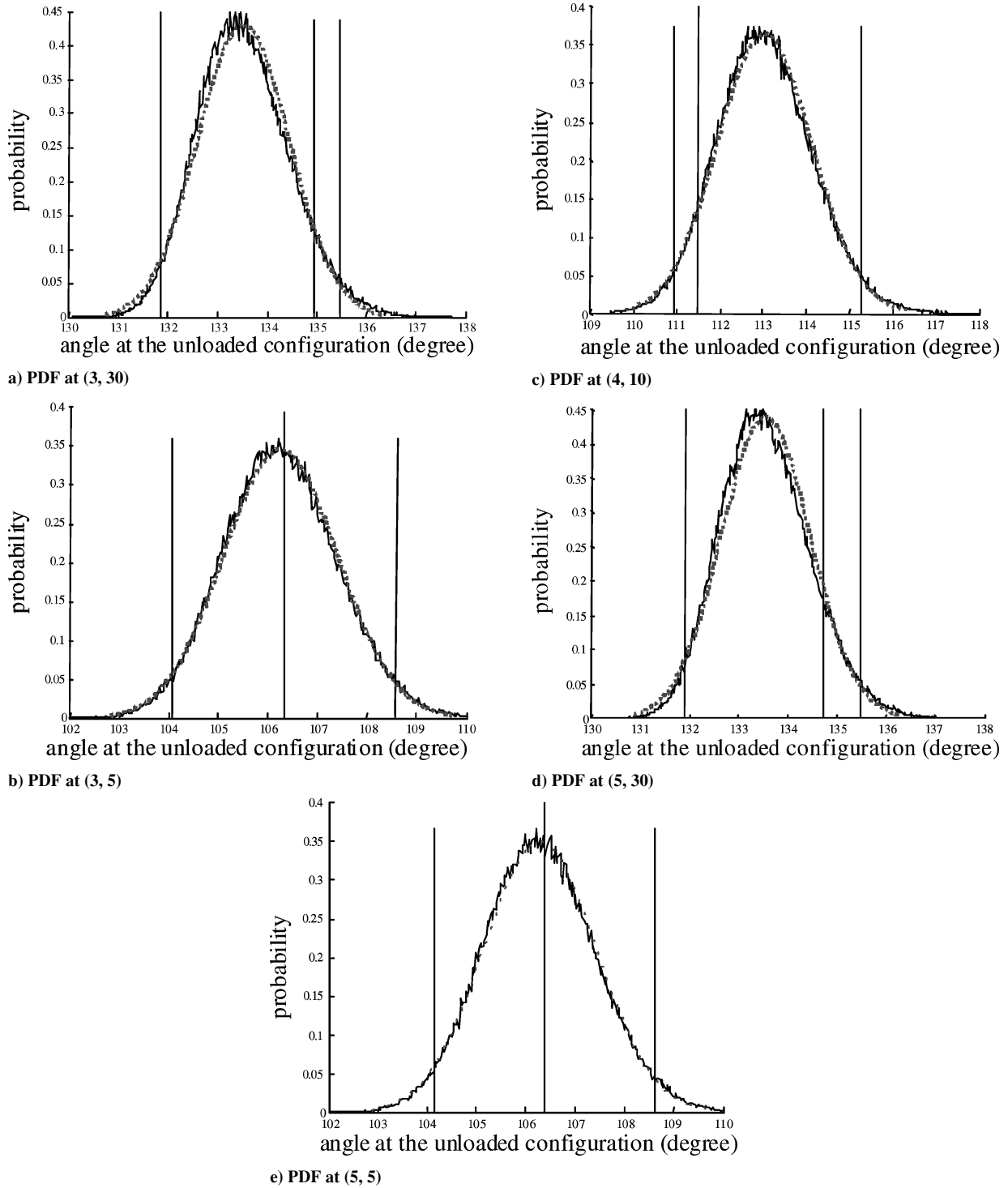


Fig. 8 PDF plots for multiple design points, model 1; lightface PDF curve, fitted normal distribution at each design point; outer vertical lines, 95% confidence level, and center vertical line, angle obtained from physical experiment at each design point.

points (3, 30), (3, 5), (4, 10), (5, 30), and (5, 5), respectively. Note that the physical experiments fall within the 95% confidence level at each design point. This means that the polynomial models considered separately at each individual design point cannot be rejected at the 95% confidence level.

Equations (2) and (3) have been used to calculate r^2 for the polynomial model. Here, r^2 for the polynomial model is 7.4462 for model 1. For the 95% confidence level, the critical value of r^2 is obtained as

$$r_{\text{critical}}^2 = l_{95\%}^2(5) = 11.07 \quad (6)$$

Because the r^2 from the polynomial model is smaller than the critical r^2 , there is not enough statistical evidence to conclude that the polynomial model is not valid. We find that, for the polynomial model, the critical confidence limit (p value) lies at about the 80% contour because r^2 for 80% for five degrees of freedom = 7.289.

Validation of model 2 (isotropic hardening law). The validation of model 2 is only illustrated for case 2, that is, for multiple design points, to demonstrate how data transformations can be applied to nonnormal response distributions. The total model uncertainty for five design points again is modeled by jointly distributed normal PDFs, and one physical experiment at each design point has been considered. It is found that the response distributions obtained

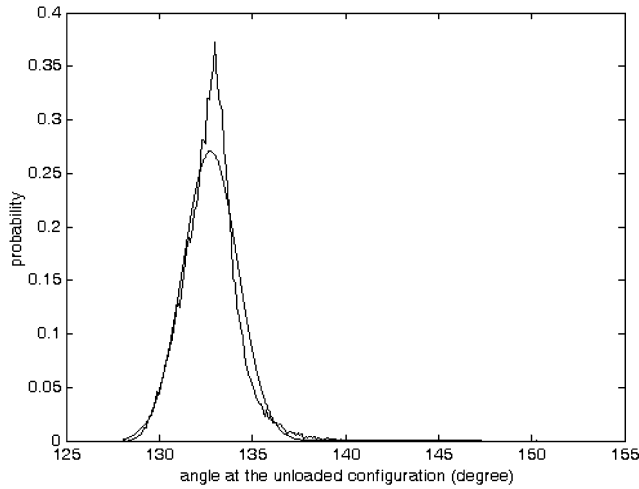


Fig. 9 PDF for multiple design points, model 2; polynomial model at design point (3,30): lightface PDF curve is fitted normal distribution.

through the RSMs are nonnormal at each design point. Note in Fig. 9 that the original distribution is right skewed with the right tail longer than the left tail. The null hypothesis for the K–S test is rejected at $\alpha = 0.05$, and the K–S statistic is 0.05 with P value of $P = 2.9408e-04$. Plots of the PDFs and the results of K–S tests at the other design points are similar to those provided for design point (3,30).

Data transformations are applied to represent the distributions in scales that are close to normal. Unfortunately, the transformations (for response only) obtained by following the existing data transformation techniques (Sec. II.D) are not satisfactory because for this problem the ratio of the largest to the smallest observation is very close to one, a condition under which the existing techniques are not applicable. We then apply transformations to both the response and the independent variables (model inputs) to overcome this difficulty. After some tests, it is found that when applying natural log transformations ($\lambda = 0$) to both dependent (i.e., the angle at the unloaded configuration) and all independent variables (i.e., the design variables and parameters) from the finite element analysis (FEA) model, the transformed distributions can be considered as normal. A polynomial response surface model is constructed using the transformed values of 243 observations from FEA at each design point:

$$\ln Z = a + \sum_i b_i \ln X_i + \sum_{i,j} c_{ij} \ln x_i \ln x_j \quad (7)$$

These RSMs are used to predict the values of transformed dependent variables and to obtain their distributions at all design points. The SSE = 0.00282179 for the new polynomial model in Eq. (7) shows that the transformed model is quite accurate. The normality check and model validation for 200,000 sample points at each design setting are carried out for the obtained transformed model.

Figure 10 provides the PDF plot of 200,000 random samples at the design point (3, 30) from the model corresponding to Eq. (7). If we compare it with Fig. 9, we can see that Fig. 10 has been improved in terms of normality and that Fig. 10 has close to a normal distribution. The K–S test is not rejected at $\alpha = 0.05$ with K–S statistic = 0.026. Plots of the PDFs and the results of K–S tests for the other design points are similar to those provided for design point (3, 30). The K–S statistics are 0.027, 0.029, 0.024, and 0.032 for design points (3, 5), (4, 10), (5, 30) and (5, 5), respectively. Note that all of the values are smaller than the critical K–S statistic, that is, 0.043. This indicates that the transformations to normal distributions are satisfactory.

With the transformed data, Eqs. (2) and (3) have been used to calculate r^2 for the transformed polynomial model 2, case 2. The r^2 for the original polynomial model 2 and for the transformed (both independent and dependent variables transformed) model 2 have been calculated to illustrate the effect of the transformation. Note that the natural logarithms of the angles from physical experiments are used in r^2 calculations. The r^2 for the transformed model 2 is

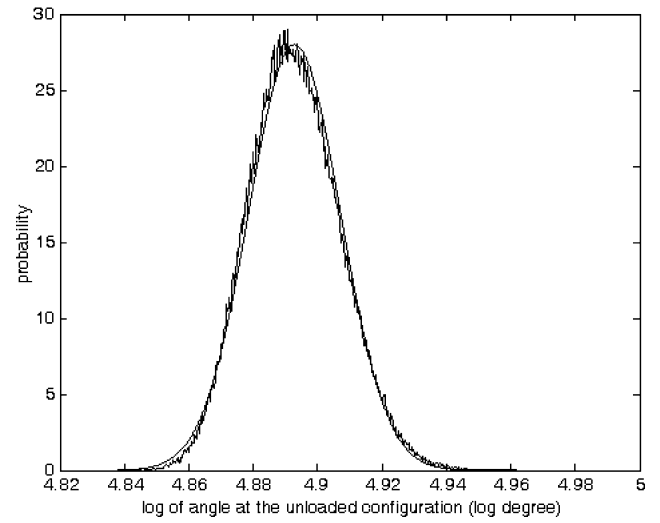


Fig. 10 Transformed PDF, model 2, after transforming independent variable and dependent variables, $\lambda = 0$, at design point (3, 30): lightface curve is fitted normal distribution.

obtained as $r^2 = 4.351$, which is smaller than the critical r^2 , that is, 11.07, for 95% confidence level. Thus, for the specified confidence level, we cannot reject the polynomial isotropic hardening model. The critical confidence limit for the transformed data, that is, for $r^2 = 4.351$, lies on about 52%. Note that, if we apply Eqs. (2) and (4) to the nonnormal response distribution without applying data transformation, the original model 2 is rejected as invalid for 95% confidence level. To make a statistically valid conclusion, data transformation needs to be applied.

Note that we have not yet considered the errors of the RSM, the experimental error, and the inaccuracy tolerance in the model validation process introduced so far. If considered, the modified confidence limit (p value) for rejecting a model is expected to be lower.

C. Measurement, RSM Errors, and Acceptable Level of Error

Following the statistical description in Sec. IV.B, the measurement error and the RSM error are added directly to the predicted values of springback angle using MCS samples. The procedure is demonstrated here only for model 1.

The mean and the variance of the RSM error are estimated as the mean and the variance of the differences between the angles obtained from the finite element simulation and those predicted with the RSM for 200 samples. The samples are obtained with optimum Latin hypercube sampling.⁴³ As the result, the normal distribution $N(-0.7120776, 0.605479)$ is used to describe the error of the polynomial model in case 1, and $N(-0.6311915, 1.0657299)$ is used to describe the error of the polynomial model in case 2.

The mean and the variance of the measurement error are obtained based on the specification of the coordinate measuring machine, represented as $N(0, 0.04193576)$. Simulation is used to obtain samples from normal distributions with the corresponding parameters as described earlier for the RSM error and the measurement error.

The simulated errors are added observation-by-observation to the samples of springback angle from MCS for cases 1 and 2. Thus, new distributions that incorporate the errors are obtained for each case. The acceptable level of error is set to ± 0.5 deg and incorporated by adjusting the result from the physical experiment. Thus, for the value of 135.52 deg obtained from the physical experiment, the model cannot be rejected if the response distribution falls to the left of the minimum acceptable value of the experiment, that is, $135.52 - 0.5 = 135.02$ deg (Fig. 11). In Fig. 11, the curve with upper tails and lower peak reflects the modified PDF, which is checked against the lower limit of acceptable error range. Note that considering different errors reduces the possibility of rejecting a model.

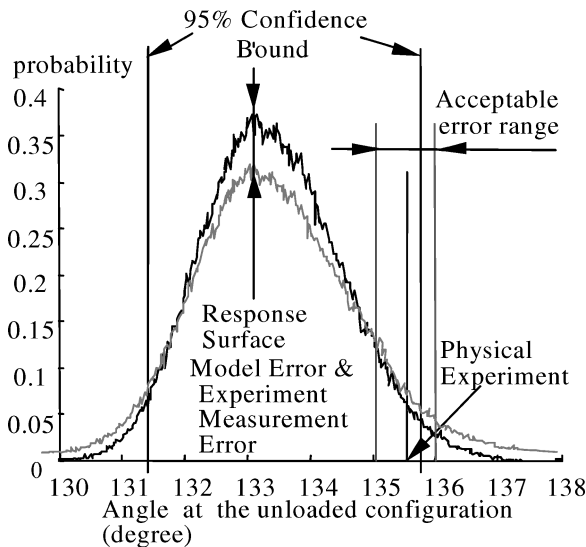


Fig. 11 Various types of errors.

The confidence limit for 135.02 deg is 87.95% for the polynomial model after the modification for case 1, single design point. The r^2 for the polynomial model is 3.67 for the minimum acceptable values of the experiments for all design points, that is, 134.4287, 106.0019, 111.1919, 134.7204, and 106.2697, for case 2, multiple design settings. As mentioned earlier, for the 95% confidence level, the critical value of r^2 is 11.07. Thus, the polynomial model for multiple design points cannot be rejected at 95% confidence level. The critical limit for the polynomial model lies on about 40% because r^2 for 40%, for five degrees of freedom = 3.6555.

V. Conclusions

An approach for model validation via uncertainty propagation using the response surface methodology is presented. The approach uses response surface methodology to create metamodels as less costly approximations of simulation models for uncertainty propagation. Our proposed model validation procedure incorporates various types of uncertainties involved in a model validation process and significantly reduces the number of physical experiments. The proposed approach can be used to provide stochastic assessment of model validity across a design space instead of a single point. The approach has been illustrated with an example of a sheet metal flanging process, for two FEMs based on the combined hardening law and the isotropic hardening law, respectively. For the FEM based on the combined hardening law, polynomial RSMs are created for both cases and confirmed to be accurate; they are used for uncertainty propagation in both cases. The critical confidence levels are identified by comparing the performance distribution obtained from uncertainty propagation with the results from the single experiments. The polynomial models have not been statistically declared as invalid if the given significant level is set at 95% for both single and multiple design points. The results are adjusted after considering the RSM error, the measurement error, and the acceptable level of error.

For the FEM model based on the isotropic hardening law, the response distribution does not follow the normal distribution. The approach suggests the use of data transformations to the polynomial model based on the isotropic hardening law. For the tested FEM based on the combined hardening law, the model cannot be statistically declared as invalid if the given significant level is set at 95% for both single and multiple design points.

Future research will be directed toward integrating the model validation approach as a part of the model selection and decision making in engineering design.

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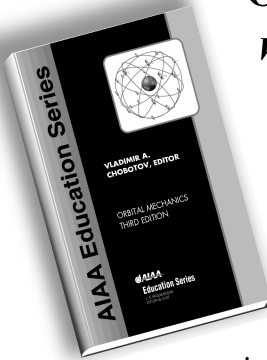
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